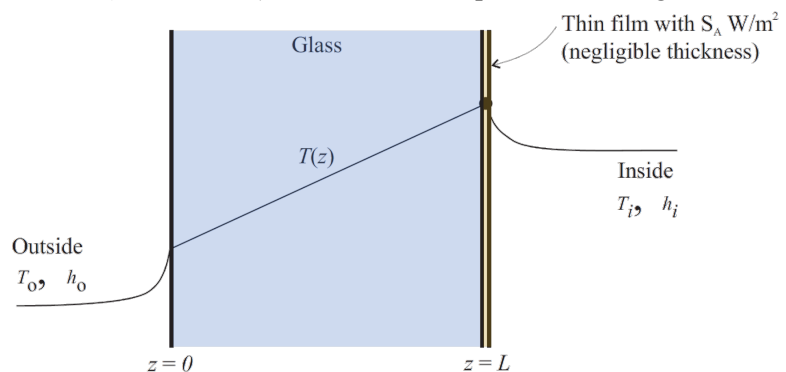


1. Heat transfer from a sphere (6 points) The smallest value that the Nusselt number can be for either natural or forced convection from a solid sphere to a fluid is 2 (see for example, equations 6.13, 6.14, or 6.20 in Module 6 notes). Can you explain in a few sentences or with a mathematical argument why this is the case? *HINT: what happens when $v_\infty = 0$?*

2. Composite with a thin element—A window defroster (14 points) Consider a composite glass slab (width L) with a very thin layer on one side (as illustrated in the figure). This geometry represents a glass car window with a thin, electrically heated film on the inside (with rate of heat production S_A in units of W/m^2) for defrosting the window in cold climates. The ambient temperature outside of the car is T_o , with a heat transfer coefficient h_o . The ambient temperature inside the car is T_i , with heat transfer coefficient h_i . When the film layer is very thin, it is reasonable to assume that the temperature in the film is constant, and equal to the glass temperature at $z = L$. Assuming steady-state operation, constant physical properties (e.g. constant k_{glass} , h_o and h_i), and that the temperature in the glass is a function of z only ($T = T(z)$), can you solve for $T(z)$ in the glass slab in terms of S_A , k_{glass} , h_o and h_i and the other parameters of the system? *Hint: the boundary condition at $z = L$ needs to be carefully considered.*

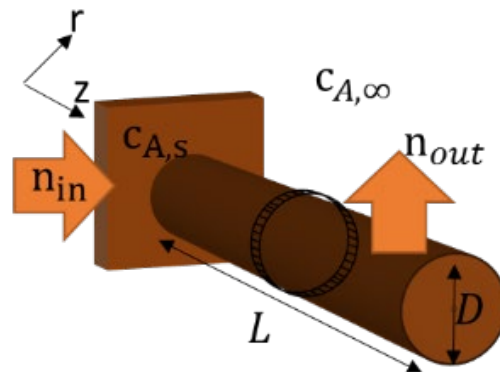


3. Hot coffee in a cup

Time for a coffee. Unfortunately, you only have a coffee cup with a broken handle so you have to hold it as shown in the (badly photoshopped) picture. You pour the hot (95°C) coffee into the cup and the cup (initially at 18°C) starts to heat up. You will have to put the cup down on the table soon, before it starts hurting your hand, but how long can you hold it? Given that the threshold for pain due to hot surfaces starts at 44°C , and using the transient solutions we developed in the course, can you estimate how long you can hold the cup before it starts hurting your hand? Here is some data and hints that will be helpful: Consider the wall of the cup as a flat plate (cartesian coordinates) with coffee on one side and a perfect insulation on the other (neglect heat transfer to your hand and the air). The thickness of the cup wall is 7 mm and the thermal conductivity of the (ceramic) cup is $6 \text{ W m}^{-1} \text{ K}^{-1}$. Its specific heat capacity and density are $850 \text{ J kg}^{-1} \text{ K}^{-1}$ and 5200 kg m^{-3} , respectively. Take the heat transfer coefficient, h , from the coffee to the cup as $500 \text{ W m}^{-1} \text{ }^\circ\text{C}^{-1}$. State any additional assumptions that you have to make to estimate your answer.



4. **Mass exchanger (6 points).** Ocean corals are animals that exchange matter with the ocean water. As corals build their calcium carbonate skeletons, they release CO₂. The corals attain a particular shape (see image) to maximize mass transfer with their environment, analogous to a “thermal dissipater” for maximizing heat exchange. In the course we derived an effectiveness factor, $\eta = \frac{\tanh N}{N}$ where $N = \sqrt{\frac{4hL^2}{Dk}}$ for a rod-type thermal dissipater. Considering a rod-shaped coral and a model for the exchange of dilute CO₂ from the coral to the ocean water analogous to the model derived in the course (i.e. ignoring the production of CO₂ in the rod, since CO₂ only enters from the base at $z = 0$) we can define an effectiveness factor for mass exchange. In this case, what would N be equal to in terms of L , D and any other parameter needed? (*Hint : check your units!*)



5. **Drying methane (10 points)**

A chemical plant requires a stream of dry methane for one of its reactors at a flow rate of 1.09 kg/s. However, the methane supply company (GAZNAT) can only supply humid methane (methane gas plus water vapor) at 1.22 bar (1.2 atm) pressure, a 30 °C dry bulb temperature and with a dew point of 21.2°C. The methane entering the reactor should be at an absolute humidity of no greater than 0.008 kg water/kg dry methane. The company asks you to design the specifications for a cooler to condition the methane at constant pressure. The humid methane will be fed into the cooler, condensing the water vapor into liquid water, and the dried methane will exit. What is the maximum temperature that the cooler should operate at ? How much heat (in Watts) needs to be removed from the methane stream? *Hint : see next page for a useful chart*

Properties of Air (at p = 1 bar):

T (°C)	ρ [kg m ⁻³]	μ [kg m ⁻¹ s ⁻¹]	ν [m ² s ⁻¹]	c_p [J kg ⁻¹ K ⁻¹]	k [W m ⁻¹ K ⁻¹]	α [m ² s ⁻¹]	β [K ⁻¹]
-20	1.396	1.622×10^{-5}	1.162×10^{-5}	1.005×10^3	0.0225	1.604×10^{-5}	3.950×10^{-3}
0	1.293	1.723×10^{-5}	1.332×10^{-5}	1.005×10^3	0.0240	1.851×10^{-5}	3.661×10^{-3}
20	1.205	1.821×10^{-5}	1.511×10^{-5}	1.006×10^3	0.0256	2.112×10^{-5}	3.411×10^{-3}
100	0.946	2.181×10^{-5}	2.306×10^{-5}	1.011×10^3	0.0313	3.271×10^{-5}	2.680×10^{-3}
200	0.746	2.563×10^{-5}	3.436×10^{-5}	1.026×10^3	0.0386	5.043×10^{-5}	2.11×10^{-3}

Properties of (liquid) Water:

T (°C)	ρ [kg m ⁻³]	μ [kg m ⁻¹ s ⁻¹]	ν [m ² s ⁻¹]	c_p [J kg ⁻¹ K ⁻¹]	k [W m ⁻¹ K ⁻¹]	α [m ² s ⁻¹]	β [K ⁻¹]
0	1000	1.690×10^{-3}	1.690×10^{-6}	4.116×10^3	0.570	1.385×10^{-7}	3.660×10^{-3}
20	997.8	9.772×10^{-4}	9.793×10^{-7}	4.076×10^3	0.605	1.487×10^{-7}	3.411×10^{-3}
100	957.1	2.748×10^{-4}	2.871×10^{-7}	4.082×10^3	0.681	1.743×10^{-7}	2.680×10^{-3}

WATER-METHANE

1.20 atm pressure

Enthalpy datum : liquid water
dry methane

0.0°C, 0.00603 atm
0.0°C, 1.00 atm

To obtain true enthalpy add enthalpy
deviation to enthalpy at saturation

